Seat No. ________________

HAJ-003-1013008

B. Sc. (Sem. III) Examination

May - 2023

Mathematics : Paper - 03 (A)

(Real Analysis)

Faculty Code : 003

Subject Code : 1013008

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions: (1) All questions are compulsory.
(2) Figure to the right indicate full marks of the question.

1 (a) Answers the following questions : **4**

(1) Define : Bounded Sequence.

(2) Define : Sequence.

(3) $\lim_{n\to\infty} \frac{1}{n^2}$ = _________.

(4) True or False : The sequence $\{(-1)^n\}$ is not convergent.

(b) Answer any **one** in brief : **2**

(1) Show that $\lim_{n\to\infty} \left[\sqrt{n+1} - \sqrt{n}\right] = 0$.

(2) Find limit points of the sequence $\{(-1)^n\}$.

(c) Answer any **one** in detail : **3**

(1) Show that every convergent sequence is bounded.

(2) Show that : $\lim_{n\to\infty} \frac{(3n+1)(n-2)}{n(n+3)} = 3$.

(d) Attempt any **one** : **5**

(1) Prove that : The limit of a convergent sequence is unique.

(2) Using definition show that : $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.

2 (a) Answers the following questions : **4**

(1) Define : Sequence of partial sum.

(2) Define : Positive term series.

(3) $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^{n} = ?$

(4) Decide the series $\sum_{n=1}^{\infty}\frac{1}{n^{\pi}}$ is convergent or divergent?

(b) Answer any **one** in brief : **2**

(1) State Logarithmic Test.

(2) State Rabbe's Test.

(c) Answer any **one** in detail : **3**

(1) Show that : If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

(2) Show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\ldots..$ is not convergent.

(d) Attempt any **one** : **5**

(1) Show that the series $\frac{1\cdot 2}{3^2\cdot 4^2}+\frac{3\cdot 4}{5^2\cdot 6^2}+\frac{5\cdot 6}{7^2\cdot 8^2}+\ldots..$ is convergent.

(2) Test for the convergence of the series

$$\sum_{n=1}^{\infty}\frac{n^2-1}{n^2+1}x^n,\ x>0.$$

3 (a) Answers the following questions : **4**

(1) Define: Curl of a vector point function.

(2) Define: Divergence of a vector point function.

(3) Define: Gradient of a scalar point function.

(4) True or False : Gradient is a vector quantity.

(b) Answer any **one** in brief : **2**

(1) If $\phi = 3x^2y - y63z^2$; find $grad\phi$ at the point (1, –2, –1).

(2) If $\overline{v} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}}$, find the value of $div\,\overline{v}$.

(c) Answer any **one** in detail : **3**

(1) If $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $grad\left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$.

(2) Prove that : The function $H = e^{-\lambda x}\left(C_1 \sin \lambda y + C_2 \cos \lambda y\right)$ satisfy the Laplace equation. Where λ, C_1 and C_2 are arbitrary constants.

(d) Attempt any **one** : **5**

(1) Let $\overline{a}$ be a constant vector. Prove that

(a) $div\left(\overline{r} \times \overline{a}\right) = 0$

(b) $curl\left(\overline{r} \times \overline{a}\right) = -2\overline{a}$

(2) Show that : $div\left(grad\ r^n\right) = n(n+1)r^{n-2}$, where $r = \sqrt{x^2 + y^2 + z^2}$.

4 (a) Answers the following questions : **4**

(1) What are the limits of x and y in the integral $\int_0^1 \int_{\sqrt{y+3}}^2 f(x, y)\, dA$.

(2) Evaluate : $\int_0^1 \int_0^2 dx\, dy$.

(3) Evaluate : $\int_0^1 \int_0^x dy\, dx$.

(4) True or False : If limits of both the variables are constants in a double integral then the region of the integration is a rectangle.

(b) Answer any **one** in brief : **2**

(1) Let $x = r\cos\theta, y = r\sin\theta$. Then find the Jacobian $J = \frac{\partial(x, y)}{\partial(r, \theta)}$.

(2) Evaluate : $\int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_0^1 r^2 \sin\theta\, dr\, d\theta\, d\phi$.

(c) Answer any **one** in detail : **3**

(1) Change the order of the integration $\int_1^4 \int_{\sqrt{y}}^2 f(x, y)\, dx\, dy$.

(2) Evaluate : $\int_0^{\frac{\pi}{2}} \int_{a(1-\cos\theta)}^{a} r^2\, dr\, d\theta$.

(d) Attempt any **one** : **5**

(1) Evaluate : $\iint_R e^{2x+3y}\, dx\, dy$ over the region bounded by $x = 0, y = 0$ and $x + y = 1$.

(2) Evaluate : $\iint_R \left(x^2 + y^2\right) dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$ by changing into polar coordinates.

5 (a) Answers the following questions : **4**

(1) Define : $\beta(m, n)$.

(2) What is the value of $\left\lceil \frac{1}{2} \right.$ = ___?___.

(3) Define: Gamma function.

(4) Find the value of $\lceil 6$.

(b) Answer any **one** in brief : **2**

(1) Show that $\beta(m, n) = \beta(n, m)$.

(2) State Stoke's Theorem.

(c) Answer any **one** in detail : **3**

(1) Evaluate : $\int_0^{\infty} \sqrt{x} e^{-3\sqrt{x}} dx$.

(2) Evaluate : $\int_0^1 x^4 \left(1 - \sqrt{x}\right)^5 dx$.

(d) Attempt any **one** : **5**

(1) Prove that : $\beta(m, n) = \dfrac{\lceil m \lceil n}{\lceil m + n}$.

(2) Using Green's Theorem evaluate $\int_C \left(x^2 y\, dx + x^2\, dy\right)$ where C is the boundary described counter clockwise of the triangle with vertices (0,0), (1,0), (1, 1).
